A Remark on the Equatorial Electrojet

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Abstract: The nighttime variation in the geomagnetic field does not show any significant equatorial anomaly. To explain this fact, characteristics of the electrical conductivity of the ionosphere in the equatorial region are studied, with special attention being given to the nocturnal condition. In the dynamo-region, the Pederson conductivity is essentially due to ions. The negative ion concentration becomes comparable with electron density during the night especially for low electron density because of the absence of photodissociation. Therefore, the ratio of the Hall conductivity to the Pederson conductivity is reduced during the night, because the Hall conductivity is proportional to electron density. This leads to a much less pronounced anomaly at the dip-equator during the night than during the daytime. With a possible model of the ionosphere, the factor of the equatorial enhancement is found to be about 13 during the daytime and only about 2 during the night. An additional description is given of the profile of the daytime electrojet.

1. Introduction

There are a number of detailed analyses of the geomagnetic data from the equatorial zone. One of the outstanding features of the equatorial anomaly in the geomagnetic field is that its Sq-variation is enhanced only during the daytime (Chapman, 1951). This was well explained by the anomalous increase in electrical conductivity of the ionosphere over the dip-equator (Hirono, 1952; Baker and Martyn, 1953). However, it has not been explained why nocturnal variations in the magnetic field do not show any significant anomaly at the equator. In this paper, some of the characteristics of the ionospheric conductivity are studied, with special attention being given to the condition during the night. With a possible model of the ionosphere, a description is also given of the profile of the equatorial electrojet.

2. General expressions for electrical conductivity

According to the study by Cowling (1945), the electrical conductivity of a partially ionized gas in which n_e electrons, n_- negative ions, and n_+ positive ions per cubic centimeter are contained is given by

$$\sigma_0 = e^2\left(\frac{n_e}{m_e}\cdot\frac{1}{\nu_e}+\frac{n_-}{m_-}\cdot\frac{1}{\nu_-}+\frac{n_+}{m_+}\cdot\frac{1}{\nu_+}\right), \tag{1}$$

where m's and ν's are masses and mean collision frequencies of an electron, a negative ion, and a positive ion, respectively. When a magnetic field is imposed, the Pederson conductivity which is perpendicular to the magnetic field is expressed by

$$\sigma_1 = e^2\left(\frac{n_e}{m_e}\cdot\frac{\nu_e}{\nu_e^2+\omega_e^2}+\frac{n_-}{m_-}\cdot\frac{\nu_-}{\nu_-^2+\omega_-^2}+\frac{n_+}{m_+}\cdot\frac{\nu^+}{\nu_+^2+\omega_+^2}\right). \tag{2}$$

In the above expression, ω's denote the gyration frequencies, and are defined by

$$\omega_e=\frac{H\,e}{m_e},\quad \omega_-=\frac{H\,e}{m_-},\quad \text{and}\quad \omega_+=\frac{H\,e}{m_+}, \tag{3}$$

where H is the strength of the magnetic field. The Hall conductivity in the direction of $\boldsymbol{H}\times\boldsymbol{E}$ is given by (Hirono, 1952 and 1953; Baker and Martyn, 1953)

$$\sigma_2 = e^2\left(\frac{n_e}{m_e}\cdot\frac{\omega_e}{\nu_e^2+\omega_e^2}+\frac{n_-}{m_-}\cdot\frac{\omega_-}{\nu_-^2+\omega_-^2}-\frac{n_+}{m_+}\cdot\frac{\omega_+}{\nu_+^2+\omega_+^2}\right). \tag{4}$$

In the ionosphere at low latitudes, the electric current can be assumed to be restricted in a sheet which is sufficiently thin compared with its horizontal dimension. In a rectangular co-ordinate system, x axis being directed to the south and y to the east, the components of electric current are given by (Hirono, 1952)

$$i_x=\sigma_{xx}E_x+\sigma_{xy}E_y\,, \tag{5}$$

$$i_y=-\sigma_{xy}E_x+\sigma_{yy}E_y\,. \tag{6}$$

In the above expressions, E_x and E_y are the components of the electrostatic field, and

$$\sigma_{xx}=\frac{\sigma_0\sigma_1}{\sigma_0\sin^2 I+\sigma_1\cos^2 I}, \tag{7}$$

$$\sigma_{xy}=\frac{\sigma_0\sigma_2\sin I}{\sigma_0\sin^2 I+\sigma_1\cos^2 I}, \tag{8}$$

and

$$\sigma_{yy}=\frac{\sigma_0\sigma_1\sin^2 I+(\sigma_1^2+\sigma_2^2)\cos^2 I}{\sigma_0\sin^2 I+\sigma_1\cos^2 I}, \tag{9}$$

where I is the dip angle of the geomagnetic field.

At and near the dip-equator, the x component of the electrostatic field must be, in general, very small compared with its y component. Therefore, the equatorial electrojet is due mostly to the equatorial enhancement in one of the tensor components of the conductivity, σ_{yy}, because it is quite natural to assume that the eastward component of the electrostatic field at the equator is not very different from that at an adjacent latitude. For this reason, we will concern ourselves only with σ_{yy} in this paper.

3. Characteristics of conductivity

The frequency of collisions of an electron or of an ion with other particles is of fundamental importance in estimating electrical conductivity. An electron collides with ions as well as neutral particles. Thus,

$$\nu_e = \nu_{e0} + \nu_{e+} + \nu_{e-}\,, \tag{10}$$

where ν_{e0}, ν_{e+} and ν_{e-} denote the collision frequencies of an electron with neutral particles, positive ions, and negative ions, respectively. The frequency of collisions of an electron with neutral particles is given by

$$\nu_{e0} = 2.5 \times 10^8 p\,, \tag{11}$$

where p is the atmospheric pressure in mmHg (Dalgarno, 1961). Collisions with ions become appreciable compared with those with neutral particles only for ion density higher than $10^6\,\mathrm{cm}^{-3}$ or in the region above 150 km. The collision frequency of an electron with neutral particles is shown as a function of altitude in Fig. 1. For an ion, the total collision frequency is given by

$$\nu_- = \nu_{-0} + \nu_{-+} + \nu_{-e} \quad \text{(for a negative ion)}, \tag{12}$$

or

$$\nu_+ = \nu_{+0} + \nu_{+-} + \nu_{+e} \quad \text{(for a positive ion)}. \tag{13}$$

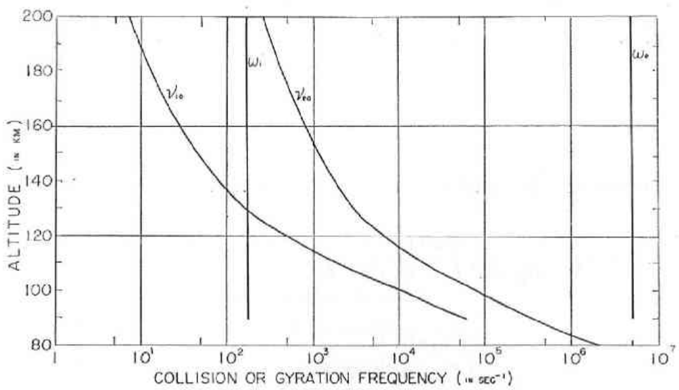

Fig. 1 The frequencies of collisions and gyrations for an electron and for an ion as functions of altitude.

The collisions with electrons can not cause any significant effect on the motion of an ion. The frequencies of collisions with ions, ν_{-+} and ν_{+-}, may also be ignored compared with those with neutral particles in the region below 200 km when ion density is lower than $10^6\mathrm{cm}^{-3}$. Thus, the total collision frequency for an ion is approximately given by ν_{-0} or ν_{+0}. Assuming that $m_+ = m_-$, we have $\nu_{-0} = \nu_{+0} = \nu_{i0}$. For an atomic oxygen ion,

$$\nu_{i0} = 6.6 \times 10^{-10} n\,, \tag{14}$$

where n is the total number density of air molecules per cm^3 (Dalgarno, 1961). One of the curves in Fig. 1 illustrates ν_{i0} as a function of altitude.

From the figure, one finds that $\nu_e/\nu_i \simeq 10^1 \sim 10^2$ in the region concerned. Since m_e/m_i is of the order of 10^{-5}, it is concluded that

$$\frac{1}{m_i\,\nu_i} \ll \frac{1}{m_e\,\nu_e}\,. \tag{15}$$

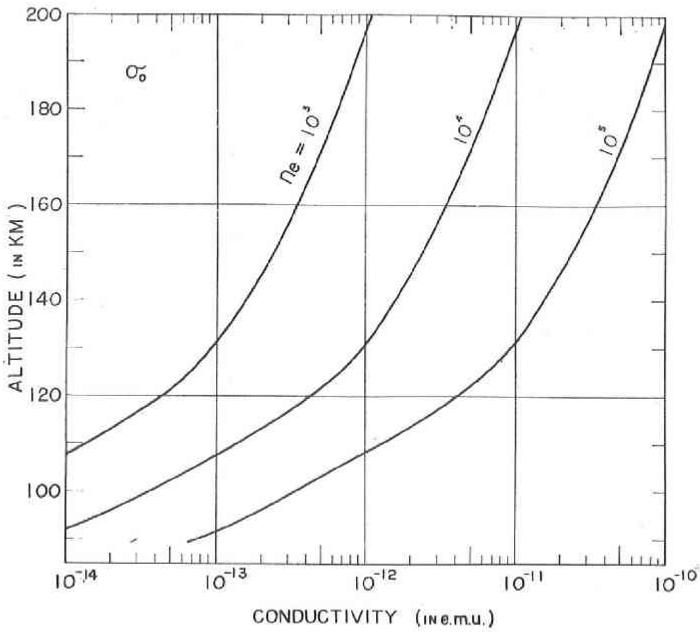

Fig. 2 The conductivity parallel to the magnetic field as a function of altitude for particular values of electron density.

In other words, the conductivity parallel to the magnetic filed, σ_0, is mostly due to electrons, and is consequently proportional to the electron density. The height variation of σ_0 is shown in Fig. 2, for the particular values of electron density.

In estimating the Pederson conductivity, the gyro-frequency for an electron or ion is an important factor as well as the collision frequency. Because $\nu/(\nu^2+\omega^2)$ is maximum when $\nu=\omega$ (ω being almost constant), its maximum is expected to be at about 80 km for electrons and at about 130 km for ions, from Fig. 1 in which ω_e and ω_i are also given. One may find from a numeral substitution that

$$\frac{1}{m_i}\,\frac{\nu_i}{\nu_i^2+\omega_i^2} \gg \frac{1}{m_e}\,\frac{\nu_e}{\nu_e^2+\omega_e^2} \tag{16}$$

in the region above 100 km. Hence it is concluded that the Pederson conductivity in the dynamo-region is essentially due to ions.

For this reason, it is convenient to express ion concentration as a function of electron density. The ratio of negative ion concentration to electron density, λ, is given by

$$\lambda = \frac{-(\gamma n+\mu+\alpha_i n_e)+\sqrt{(\gamma n+\mu+\alpha_i n_e)^2+4\,\alpha_i\,\beta\,n_e\,n}}{2\,\alpha_i\,n_e}, \tag{17}$$

where n is neutral particle concentration, γ, μ, α_i and β are the rate coefficients for collisional detachment, photodetachment, ionic neutralization, and attachment, respectively (Kamiyama, 1959). Using the revised values for these coefficients listed below, the computed result for λ at various altitudes both in daytime and nighttime conditions are shown in Fig. 3 as functions of electron density. From the figure, one may say that negative ion concentration is comparable with electron density in the dynamo-region during the night for lower values of electron density, whereas the former is negligible compared with the latter during the daytime.

Table 1 Rate Coefficients

Process	Coefficient	Reference
Ionic neutralization, α_i	$1\times10^{-7}\mathrm{cm^3sec^{-1}}$	Sayers, 1962
Attachment, β	$1.4\times10^{-15}\mathrm{cm^3sec^{-1}}$	Branscomb, et al, 1958
Photodetachment, μ	$1.4\ \mathrm{sec^{-1}}$ per negative ion	Branscomb, et al, 1958
Collisional detachment, γ	$1\times10^{-16}\mathrm{cm^3sec^{-1}}$	Massey, 1937

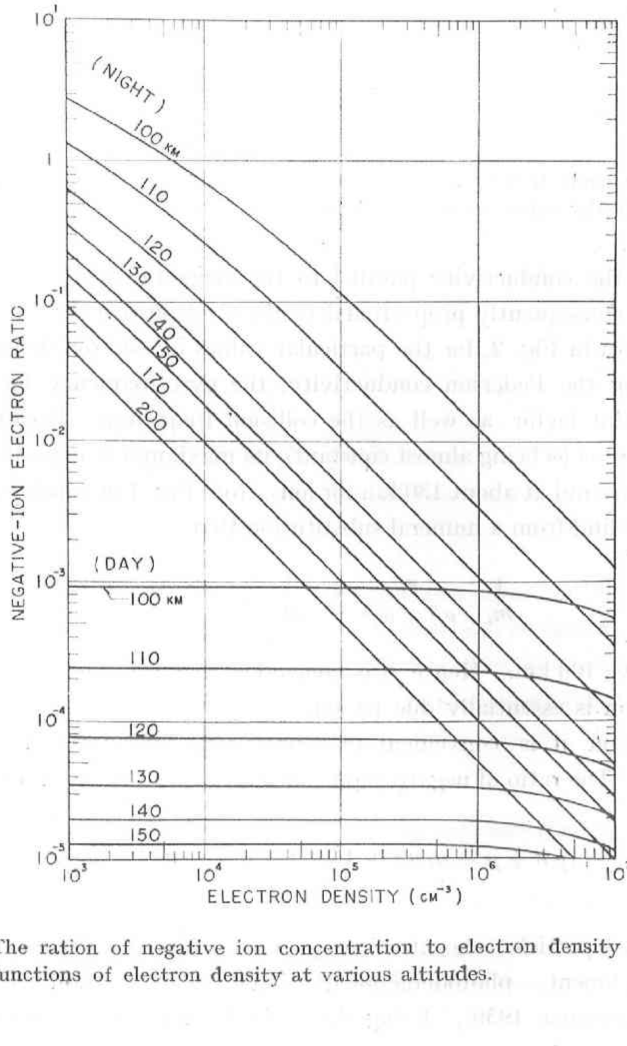

Fig. 3 The ration of negative ion concentration to electron density shown as functions of electron density at various altitudes.

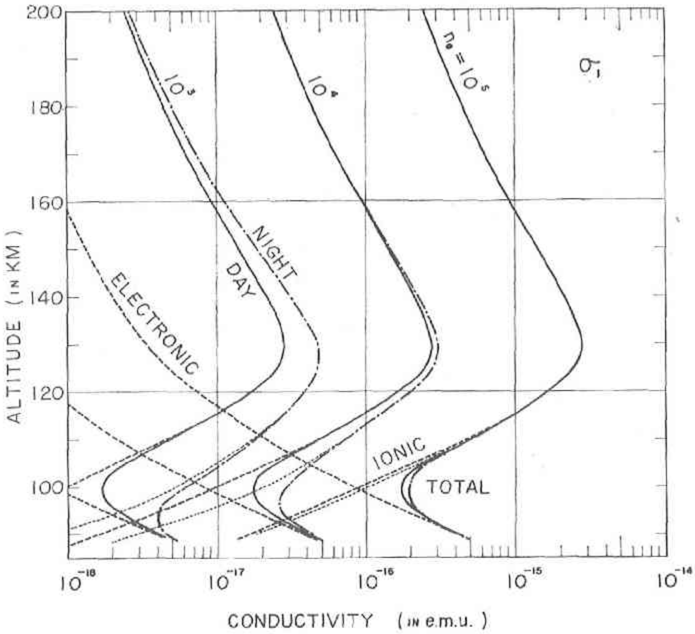

Fig. 4 The Pederson conductivity, σ_1 as functions of altitude. The total contribution both from electrons and ions is shown as full lines for daytime and as dashed-and-dotted lines for nighttime. The dashed lines illustrate the electronic or ionic contribution separately for daytime, and the dotted lines show the ionic contribution for nighttime.

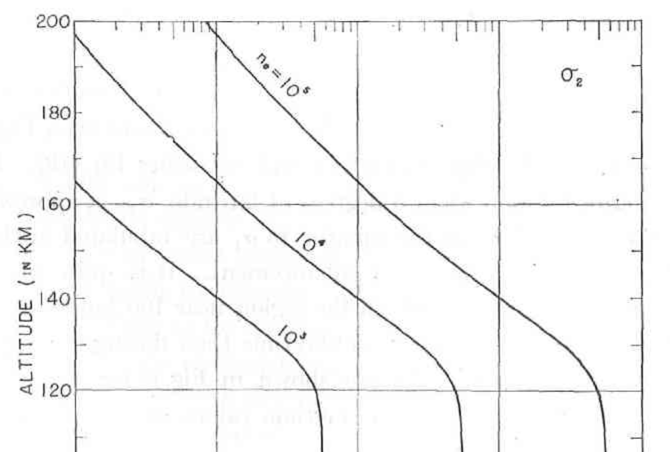

Fig. 5. The Hall conductivity, σ_2, as functions of altitude for particular values of electron density.

As mentioned previously, the Pederson conductivity, σ_1, in the dynamo-region is mostly due to ions. Hence, we can conclude that σ_1 is proportional to $(1+2\lambda)n_e$, where λ is almost inversely proportional to electron density, n_e, and is comparable with unity during the night for lower values of n_e. The computed results for σ_1 are shown in Fig. 4 as functions of height for special values of electron density. The total contribution both from electrons and ions is drawn by the full lines for daytime and by the dashed-and-dotted lines for nighttime. The dashed lines illustrate the electronic or ionic contribution separately for daytime, and the dotted lines show the ionic contribution in the nighttime condition. Thus, the reduction in σ_1 during the night is considerably less than that in electron density especially for its lower values.

In the expression for the Hall conductivity σ_2 (Eq. 4), if we assume $m_+=m_-$, it is easily seen that σ_2 is exactly proportional to electron density n_e. The results for σ_2 are shown in Fig. 5 for the special values of electron density. It is said that σ_2 is almost constant in the region below 120 km for a constant electron density. Therefore the maximum of the actual distribution of σ_2, in the ionosphere is expected to be at a altitude where electron density in the E layer attains to its maximum.

4. The conductivity at the dip-equator

The tensor component, σ_{yy}, of the conductivity is reducible to

$$\sigma_{yy} = \sigma_1 + \frac{{\sigma_2}^2}{\sigma_1} \tag{18}$$

at the dip-equator. From Figs. 4 and 5, the Hall Conductivity σ_2 is found to be larger than the Pederson conductivity σ_1 in the dynamo-region. This is much more pronounced during the daytime than during the night, because σ_1 is proportional to $(1+2\lambda)n_e$ whereas σ_2 decreases in proportion as n_e. For example, the following values listed in Table 2 for σ_0, σ_1, and σ_2 in the particular conditions selected can be found from Figs. 2, 4, and 5. With these values, σ_{yy} at the dip-equator is found by using Eq. (18). In the region apart from the equator by more than 5 degrees of latitude, σ_{yy} is approximately given by σ_1. Hence, the ratios of σ_{yy} at the equator to σ_1 are tabulated in the last column in Table 2 to characterize the equatorial enhancement. It is quite obvious that the enhancement is much more remarkable in the region near 100 km-level than in higher region, and is more pronounced during the daytime than during the night. The computed results for σ_{yy} by using Eq. (9) are shown in Fig. 6 for equatorial latitudes. In this figure with logarithmic scale, the daytime values are shown on the right-hand

Table 2. Sampled values of conductivity (in e.m.u.)

Condition		σ_0	σ_1	σ_2	σ_{yy}	σ_{yy}/σ_1
Day $n_e=10^5\text{cm}^{-3}$	100 km	3.0×10^{-13}	1.8×10^{-16}	5.5×10^{-15}	1.7×10^{-13}	944
	125 km	5.0×10^{-12}	2.0×10^{-15}	5.0×10^{-15}	1.5×10^{-14}	7.5
Night $n_e=10^3\text{cm}^{-3}$	100 km	3.0×10^{-15}	6.5×10^{-18}	5.5×10^{-17}	4.7×10^{-16}	72
	125 km	5.0×10^{-14}	4.5×10^{-17}	5.0×10^{-17}	1.0×10^{-16}	2.2

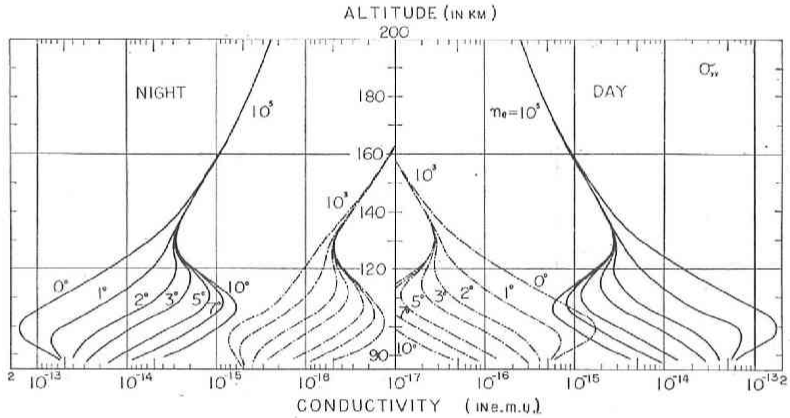

Fig. 6 The results for σ_{yy} shown as functions of altitude for particular values of electron density. The nighttime values are given on the left-hand half of the figure with the abscissa directed toward the left.

side of the center line, and the nighttime values are on the left-hand half with the abscissa directed toward the left. An asymmetry about the center line is obvious for the low value of n_e. If there were the electron density of the order of $10^5\ \mathrm{cm}^{-3}$ in the dynamo-region during the night, geomagnetic variations at the equator would be intensified as in daytime.

It is known that the height of the E layer is generally higher during the night than during the daytime. Therefore, the electron density in the region (near 100 km) where σ_{yy} would be enhanced at the equator may be considerably low during the night. Hence, the contribution from the enhanced conductive region to the integration with respect to height may be much smaller during the night than during the daytime. This may be an additional reason why the equatorial intensification of geomagnetic variations is remarkable only during the daytime.

With the vertical distribution of electron density in the ionosphere as shown in Fig. 7 (left) which is based on the rocket observation (Jackson and Seddon, 1958), a profile of the equatorial electrojet during the daytime is shown in Fig. 7 (right). It is seen from the figure that the electrojet flows in the equatorial belt with a half-width of $2 \sim 3°$ latitude. It is also pointed out that the electrojet locates below the horizontal conductive sheet which may not vary remarkably with latitude. This is consistent with the model deduced by Onwumechilli (1959) from his analysis of the geomagnetic data. In a suitable condition of the electron distribution (with a pronounced maximum at a level between 100 km and 110 km), the doubly stratified current sheets could be expected near the dip-equator as observed by using the rocket-magnetometer (Cahill, 1959). Apart from the equator, the east-west component of the ionospheric current depends not only on σ_{yy} but also on σ_{xy} which, in general, attains to its maximum at a level between 100 and 110 km where electron density is maximum. Therefore, there could possibly be two current sheets at middle latitudes, one being located between 100 and 110 km, and the other between

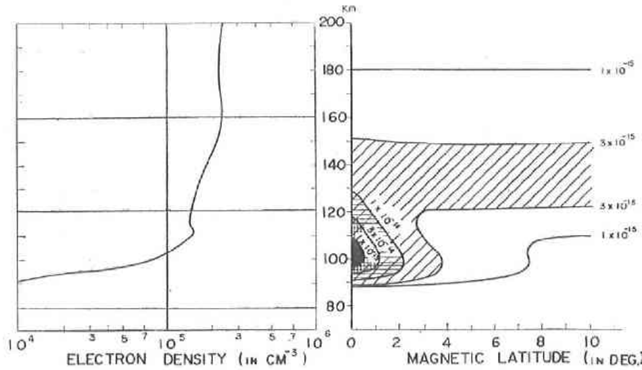

Fig. 7 A profile of the equatorial electrojet (right) deduced from a model of the ionosphere during the daytime (left).

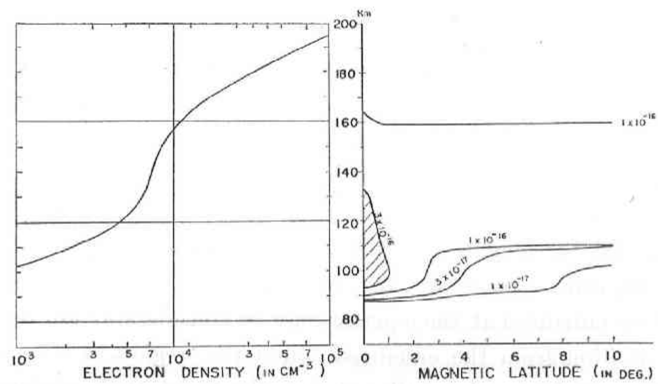

Fig. 8 A profile of the conductive sheet in the equatorial region (right) deduced from a possible distribution of electron density during the night (left).

120 and 150 km. It is also interesting to note that the equatorial Es-ionization appears most frequently in the equatorial belt with the same width as that of the electrojet (Wright and Hibberd, 1963).

Assuming that the effective recombination coefficient is given by $\alpha_e+\lambda\alpha_i$ where α_e denotes the coefficient of an appropriate dissociative recombination ($3\times10^{-9}\,cm^3\,sec^{-1}$), and that the electron density at the sunset is one fifth of that shown in Fig. 7, we have a electron density distribution as shown in Fig. 8 (left) 8 hours after the sunset. With this electron distribution, the profile of the conductive sheet near the equator is illustrated in Fig. 8 (right). Thus, only a slight enhancement is seen at the equator during the night. The latitudinal variation of the integration of σ_{yy} with respect to height up to 200 km is shown in Fig. 9 both in the day and night conditions. The factor of the equatorial enhancement is about 13 during the daytime and only about 2 during the night. Although the profiles of the conductive sheet shown in Figs. 7

and 8, and consequently the result shown in Fig. 9 may be modified for another distribution of electron density, the essential tendency described in this paper will not be changed.

5. Conclusion

A detailed study of characteristics of the ionospheric conductivity leads to the following conclusions: (1) The electrical conductivity parallel to the geomagnetic field (σ_0) is mostly due to electrons. Therefore, σ_0 is proportional to electron density. (2) The Pederson conductivity (σ_1) is mainly given by the ionic contribution in the region above 100 km, and by the electronic contribution in the region below this level. Therefore, σ_1 in the dynamo-region is not proportional to electron density but to ion density. (3) Ion density is given by $(1+2\lambda)n_e$, where λ is the ratio of negative ion density to electron density. The ratio, λ, is negligible compared with unity during the daytime, whereas it is comparable with unity in the dynamo-region during the night and is almost inversely proportional to electron density. (4) Consequently, σ_1 decreases much less than electron density does during the night. (5) The Hall conductivity (σ_2) must be proportional to electron density. (6) From these fundamental characteristics of the ionospheric conductivity, we have deduced that σ_2/σ_1 in the dynamo-region is very large during the daytime, whereas in the nighttime condition, it is, in general, about one tenth of the daytime value. (7) This fact gives rise to a anomalous enhancement of σ_{yy} at the dip-equator only during the daytime. The intensification factor at the equator is probably more than 10 during the daytime, whereas it is only about 2 during the nighttime. (8) With a possible model of the ionosphere, the equatorial electrojet during the daytime is shown to de located below the uniform sheet which may not vary so much with latitude, the electrojet being near 100 km-level and the uniform sheet between 120 and 150 km. (9) In the suitable condition of the electron distribution, two sheets of electric current could be expected, the lower being between 100 and 110 km, and the upper in the region above 120 km. (10) The half-width of the equatorial electrojet is about 3° of latitude.

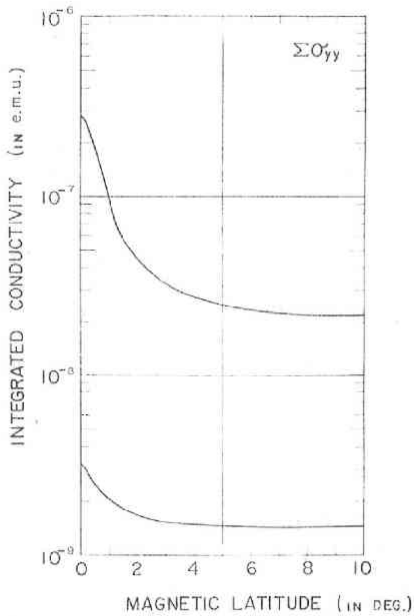

Fig. 9 Latitudinal variation in the integration of σ_{yy} with respect to height up to 200 km. The upper curve is based on Fig. 7 for daytime, and the lower on Fig. 6 for nighttime.

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